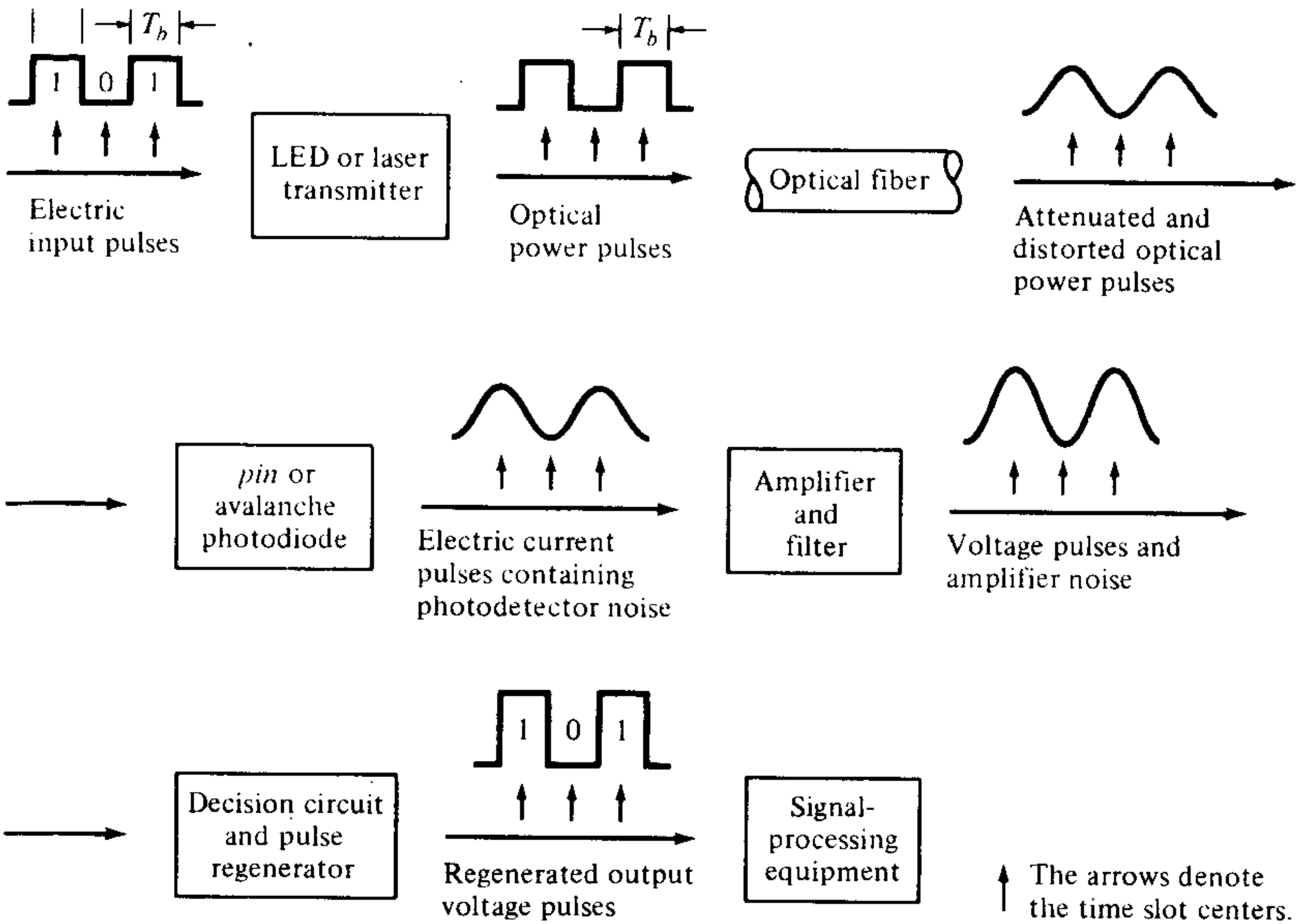
OPTICAL FIBER COMMUNICATION

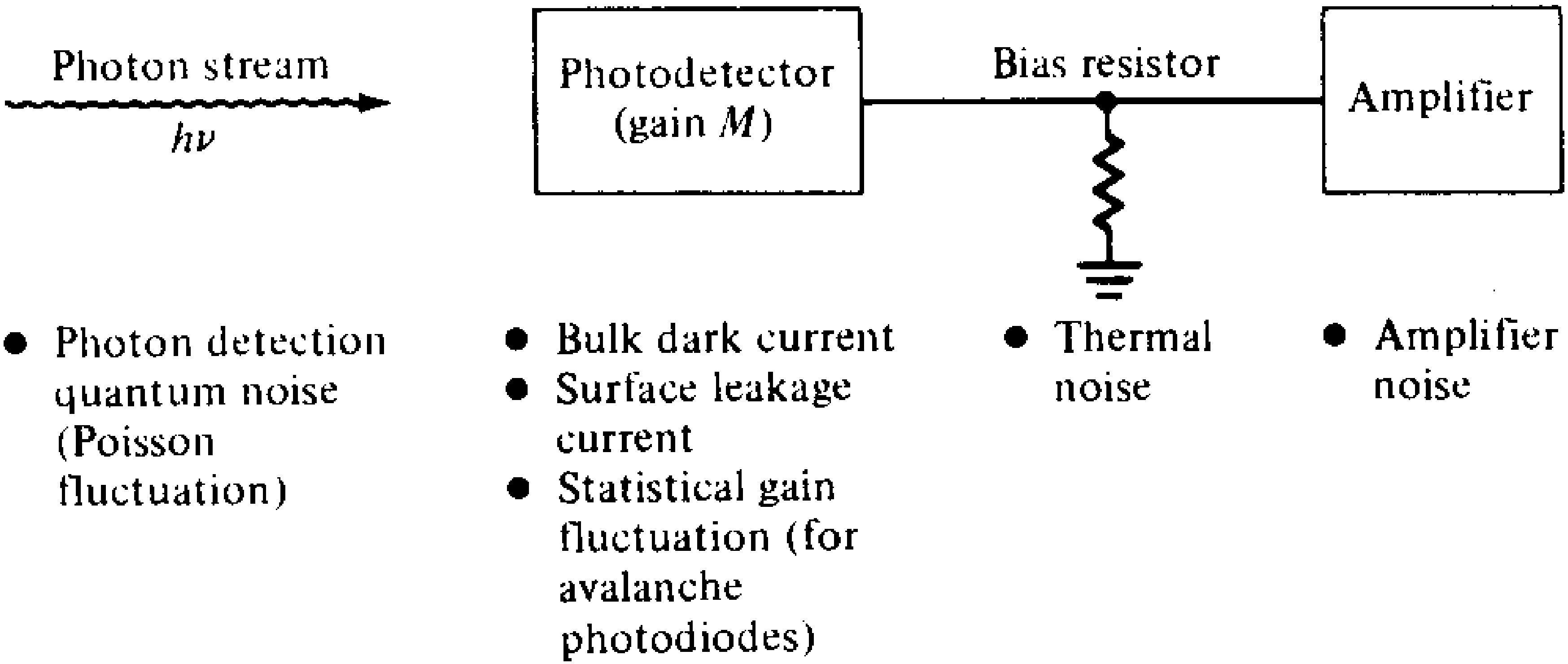
PART III:-
DIGITAL RECEIVERS

- Digital Receivers
- Probability of error
- Quantum Limit
- Shot noise
- Noise Penalty
- Pre-amplifier types

DIGITAL RECEIVER



Digital Receiver



- Analog system—Signal to rms noise ratio
- Digital system– Average error probability

Digital Receiver-
Photon detection quantum noise

- Due to random arrival rate of signal photons.
- Makes primary photocurrent a time varying Poisson Process.
- If detector illuminated by optical signal p(t), then average number of electron-hole pair generated in time τ is --

$$\bar{N} = \frac{\eta}{h\nu}\int_0^{\tau} P(t)\,dt = \frac{\eta E}{h\nu}$$

- η is detector quantum efficiency.

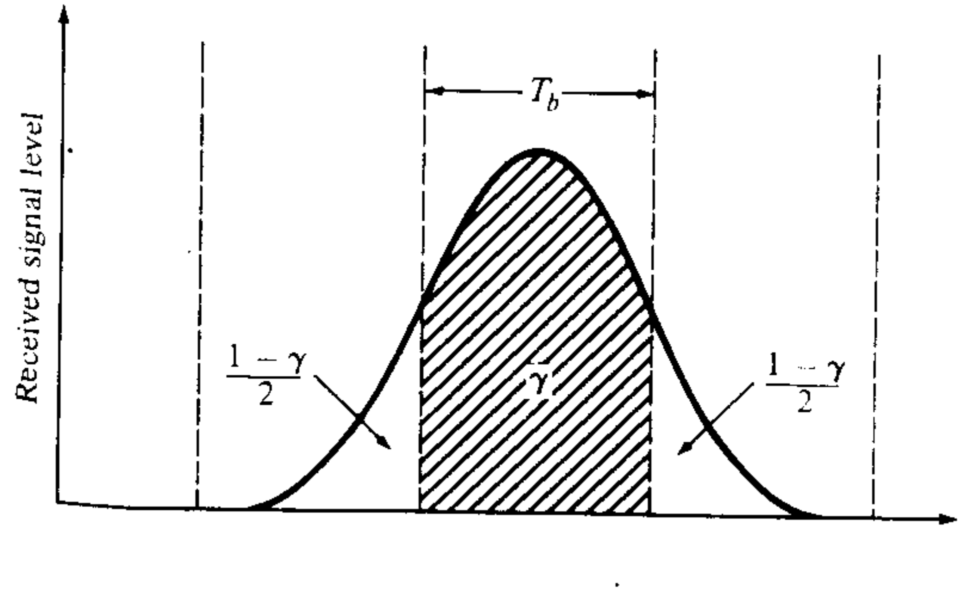
Digital Receiver-
Photon detection quantum noise

- Actual number of electron-hole pair n fluctuates from average according to Poisson distribution.

$$P_r(n) = \bar{N}^n \frac{e^{-\bar{N}}}{n!}$$

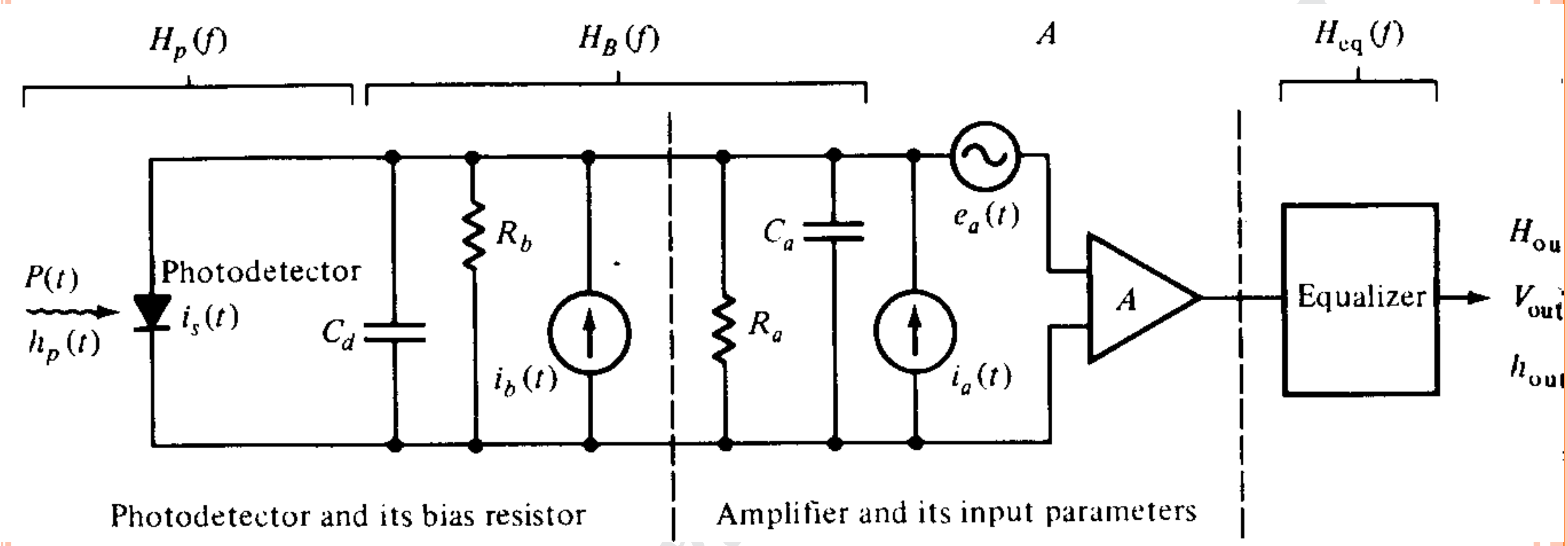
where $P_r(n)$ is the probability that n electrons are emitted in an interval τ.

Digital Receiver



- As the pulse progresses, it spreads and enters into adjacent time slots causing ISI.
- Major part γ in desired slot while rest spreads.

Digital Receiver



- $h_p(t)$ - input pulse shape
- $H_p(\omega)$ – Fourier transform of
- $H_B(\omega)$ – transfer function of bias circuit
- $H_{eq}(\omega)$ – transfer function of equalising circuit
- A – Gain of amplifier

Digital Receiver

- η – Quantum efficiency of photo detector
- C_d – Photodiode capacitance
- R_b – Detector bias resistance
- R_a II C_a – Amplifier input impedance
- C_a – Amplifier shunt capacitance
- $i_b(t)$ – Thermal noise current generated by R_b
- $i_a(t)$ – Thermal noise current generated by R_a
- $v_a(t)$ – Thermal noise voltage of amplifier channel
- Input voltage develops across R_a
- Two amplifier noise sources $i_a(t)$, $v_a(t)$
- One detector noise source $i_b(t)$ due to bias resistor.

DIGITAL RECEIVER

- All noises Gaussian, have flat spectral response (white noise), uncorrelated, statically independent .
- Occurrence of one doesn't effect occurrence of other.
- Input pulse train is -

$$P(t) = \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

- b_n – Amplitude of n^{th} pulse
- h_p - received Pulse shape
- T_b – Bit period
- b_n can be b_{on} or b_{off}
- $h_p(t)$ normalized to have unit area.

$$\int_{-\infty}^{\infty} h_p(t)\,dt = 1$$

DIGITAL RECEIVER

- Mean output current from detector -

$$\langle i(t)\rangle = \frac{\eta q}{h\nu} MP(t) = \mathscr{R}_0 M \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

where $\mathscr{R}_0 = \eta q / h\nu$ is the photodiode responsivity

- Current amplified and filtered to produce mean voltage at output of equalizer -

$$\langle v_{\text{out}}(t)\rangle = A\mathscr{R}_0 MP(t) * h_B(t) * h_{\text{eq}}(t)$$

$$= \mathscr{R}_0 GP(t) * h_B(t) * h_{\text{eq}}(t) \qquad G = AM$$

- $h_B(t)$ and $h_{eq}(t)$ are impulse responses of bias and equalizer circuits.

Digital Receiver

$$h_B(t) = F^{-1}[H_B(f)] = \int_{-\infty}^{\infty} H_B(f) e^{j2\pi f t}\, df$$

$$H_B(f) = \frac{1}{1/R + j2\pi f C}$$

$$\frac{1}{R} = \frac{1}{R_a} + \frac{1}{R_b}$$

$$C = C_a + C_d$$

- The mean output voltage from equalizer is -

$$\langle v_{\text{out}}(t) \rangle = \sum_{n=-\infty}^{\infty} b_n h_{\text{out}}(t - nT_b)$$

where

$$h_{\text{out}}(t) = \mathscr{R}_0 G h_p(t) * h_B(t) * h_{\text{eq}}(t)$$

$$H_{\text{out}}(f) = \int_{-\infty}^{\infty} h_{\text{out}}(t) e^{-j2\pi f t}\, dt = \mathscr{R}_0 G H_p(f) H_B(f) H_{\text{eq}}(f)$$

Probability of error

$$\text{BER} = \frac{N_e}{N_t} = \frac{N_e}{Bt}$$

N_e – errors occurring in time t
N_t – total pulses transmitted in time t
B – bit rate = $1/T_b$

- Assuming the noise has Gaussian probability density function with zero mean.
- Noise voltage n(t) sampled at any arbitrary time t,
- The probability that the measured sample n(t) falls in range n to $n+dn$ is -

$$f(n)\,dn = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n^2/2\sigma^2}\,dn$$

where σ^2 is the noise variance and $f(n)$ is the probability density function.

PROBABILITY OF ERROR- CASE I – ‘0’ IS BEING SENT.

- Let transmitted pulses are ‘0’ and ‘1’, $v_{th} = V/2$
- Transmitted = ‘0’
- Received r(t) = n(t) = v
- Probability that it be detected as ‘1’ is probability that v lied between V /2and ∞.

$$P_0(v_{th}) = \int_{V/2}^{\infty} p(y|0)\,dy = \int_{V/2}^{\infty} f_0(y)\,dy$$

$$= \int_{V/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-v^2/2\sigma^2}\,dv$$

where the subscript 0 denotes the presence of a 0 bit.

Probability of error- case II – ‘1’ is being sent.

- Transmitted = ‘1’
- Received r(t) = V + n(t) = v
- n(t) = v - V
- Probability that it be detected as ‘0’ is probability that v lied between - ∞and V /2.

$$f_1(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(v-V)^2/2\sigma^2}.$$

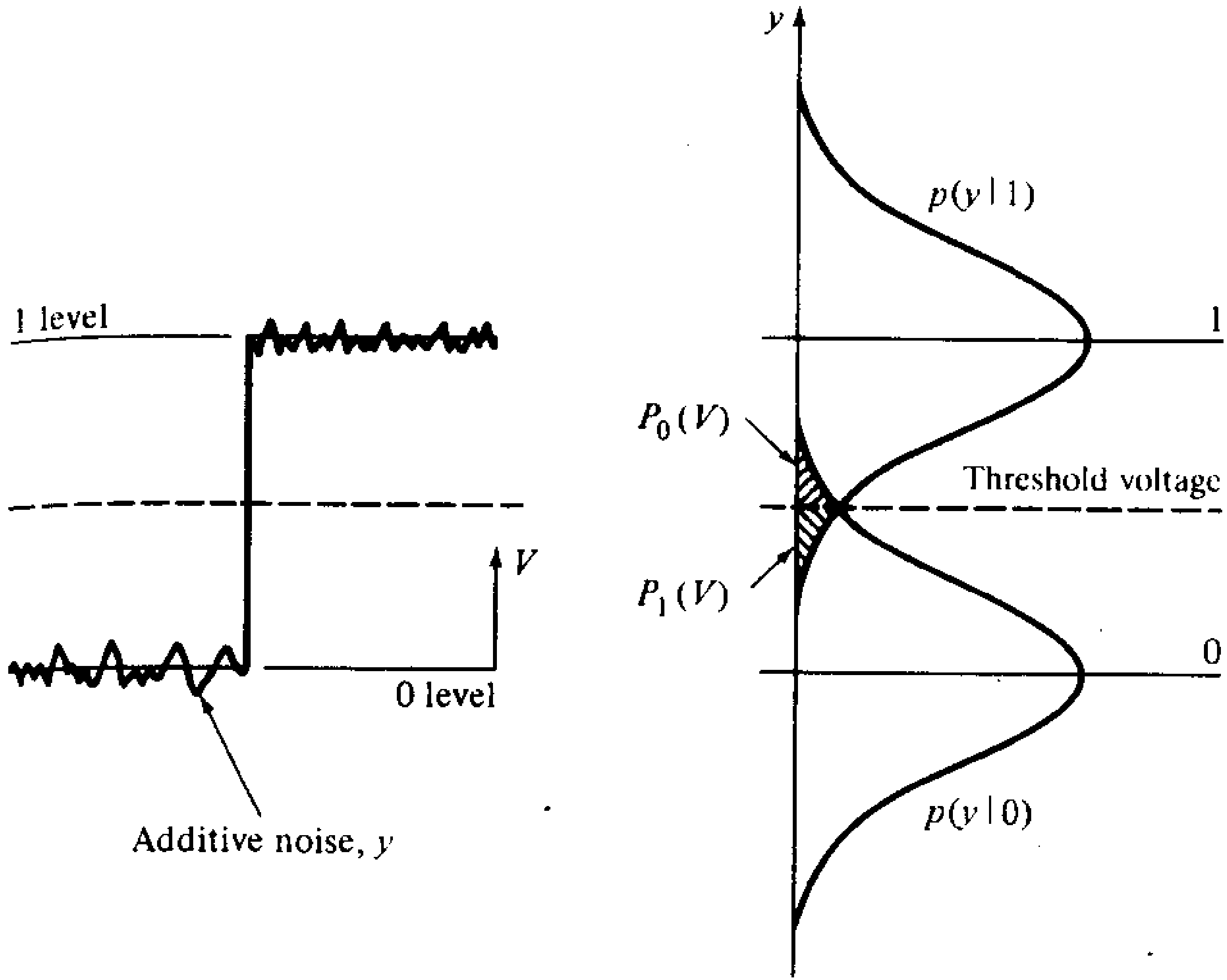
$$P_1(v_{th}) = \int_{-\infty}^{V/2} p(y|1)\, dy = \int_{-\infty}^{V/2} f_1(v)\, dv$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{V/2} e^{-(v-V)^2/2\sigma^2}\, dv$$

Total Probability of error-

- $P_e = a\, p_0(v) + b\, p_1(v)$
- Assuming '0' and '1' are equiprobable, $a = b = 0.5$.
- $P_e = ?$
- Looking at distribution $p_0(v)$ and $p_1(v)$ are identical.
- Integrating double of one part -

Total Probability of error-



Total Probability of Error-

$$P_e = \int_{V/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-v^2/2\sigma^2} dv$$

- $v/\sqrt{2}\,\sigma = x, \quad dv/\sqrt{2}\,\sigma = dx$
- Limits = ? P_e = ?
- Expression not integrable. Tabulated as erfc(x) or erf(x).
- Comparing with erfc(x)
- $\text{erfc}(x) = 2/\sqrt{\pi} \int_x^{\infty} e^{-y^2} dy$

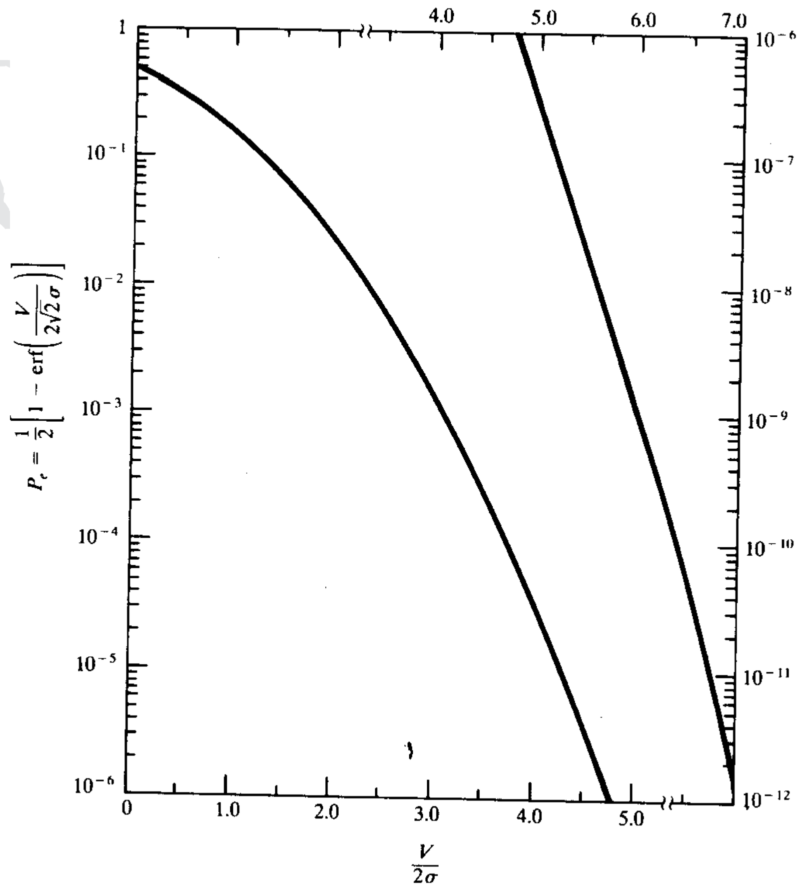
$$P_e = \tfrac{1}{2}\,\text{erfc}(V/(2\sqrt{2}\,\sigma))$$

TOTAL PROBABILITY OF ERROR-

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{V}{2\sqrt{2}\,\sigma}\right)\right]$$

$$\left(\frac{S}{N}\right)_{\mathrm{dB}} = 20 \log \frac{V}{\sigma}$$

By doubling V, BER decreases by 10^4.



QUANTUM LIMIT TO DETECTIO

$$P_r(n) = \bar{N}^n \frac{e^{-\bar{N}}}{n!}$$

- Ideal photo detector having unity quantum efficiency and no dark current.
- No e-h pair generated in absence of optical pulse.'0'
- Possible to find minimum received optical power required for specific BER performance in digital system.
- Called Quantum limit.

QUANTUM LIMIT TO DETECTIO

$$P_r(n) = \bar{N}^n \frac{e^{-\bar{N}}}{n!}$$

- Optical pulse of energy E falls on photo detector in time interval τ.
- During transmission signal if too low to generate any e-h pair and detected as 0.
- Then for error probability $P_r(0)$, there exists a minimum energy E at wavelength λ, to be detected as 1.
- Probability that n=0 electrons are emitted in interval τ-

$$P_r(0) = e^{-\bar{N}}$$

QUANTUM LIMIT TO DETECTION - PROBLEM

$$\bar{N} = \frac{\eta}{h\nu}\int_0^{\tau} P(t)\, dt = \frac{\eta E}{h\nu}$$

- Digital fiber optic link operating at wavelength 850nm requires maximum BER of 10^{-9}. Find quantum limit and minimum incident power P_o that must fall on photo detector , to achieve this BER at data rate of 10Mbps for simple binary level signaling system. Quantum efficiency is 1.
- Solution –for maximum BER,--

$$P_r(0) = e^{-\bar{N}} = 10^{-9}$$

$$\bar{N} = 9 \ln 10 = 20.7 \simeq 21.$$

$$E = 20.7 \frac{h\nu}{\eta}$$

Quantum Limit to detection - problem

- Minimum incident power that must fall on photo detector P_o --- $E = P_o \tau$
- Assuming equal number of 0 and 1, $1/\tau = B/2$

$$P_0 = 20.7 \frac{hcB}{2\lambda}.$$

$$= \frac{20.7(6.626 \times 10^{-34} \text{J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})(10 \times 10^6 \text{ bits/s})}{2(0.85 \times 10^{-6} \text{ m})}$$

$$= 24.2 \text{ pW}$$

or, when the reference power level is one milliwatt,

$$P_0 = -76.2 \text{ dBm}$$

Receiver Noises

- Noise voltage $v_N(t)$ causes v_{out} to deviate from mean or average $< v_{out} >$

$$v_{out}(t) = \langle v_{out}(t) \rangle + v_N(t)$$

$$v_N^2(t) = v_s^2(t) + v_R^2(t) + v_I^2(t) + v_E^2(t)$$

- $v_s(t)$ – Quantum or shot noise due to random multiplied poisson nature of photocurrent $i_s(t)$.
- $v_R(t)$ – thermal noise due to bias resistor R_b.
- $v_I(t)$ –noise due to amplifier input noise
- $v_E(t)$ –noise due to amplifier due to $e_a(t)$.

• Calculating the three thermal noise currents at the output of equalizer :-

The thermal noise of the load resistor R_b

$$\langle v_R^2(t) \rangle = \frac{4k_B T}{R_b} B_{bae} R^2 A^2$$

B_{bae} – Noise equivalent bandwidth of bias ckt, amplifier and equalizer

$$B_{bae} = \frac{1}{|H_B(0) H_{eq}(0)|^2} \int_0^\infty |H_B(f) H_{eq}(f)|^2 \, df$$

$$= \frac{1}{|H_{out}(0)/H_p(0)|^2} \int_0^\infty \left| \frac{H_{out}(f)}{H_p(f)} \right|^2 df$$

$$\langle v_I^2(t) \rangle = S_I B_{bae} R^2 A^2$$

$$\langle v_E^2(t) \rangle = S_E B_e A^2$$

where S_I is the spectral density of the amplifier input noise current source (measured in amperes squared per hertz), S_E is the spectral density of the amplifier noise voltage source (measured in volts squared per hertz), and

$$B_e = \frac{1}{|H_{eq}(0)|^2} \int_0^\infty |H_{eq}(f)|^2 \, df$$

$$= \frac{R^2}{|H_{out}(0)/H_p(0)|^2} \int_0^\infty \left| \frac{H_{out}(f)}{H_p(f)} \left(\frac{1}{R} + j2\pi fC \right) \right|^2 df$$

Shot Noises-

$$\langle v_s^2(t) \rangle = 2q\langle i_0 \rangle \langle m^2 \rangle B_{bae} R^2 A^2$$

$\langle m^2 \rangle$ is the mean square avalanche gain

- Shot noise in bit period T_b is shot noise contribution from a pulse within that period as well as from all other pulses outside that period.
- Worst case shot noise when all neighboring pulses are '1'.
- Greatest ISI.
- Hence mean unity gain photocurrent over T_b for 1 pulse -

$$\langle i_0 \rangle_1 = \sum_{n=-\infty}^{\infty} \frac{\eta q}{h\nu} b_{on} \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} h_p(t - nT_b)\, dt$$

Shot Noises – '1' with all neighbor '1's

$$= \frac{\eta q}{h\nu} \frac{b_{\text{on}}}{T_b} \int_{-\infty}^{\infty} h_p(t)\, dt$$

$$= \frac{\eta q}{h\nu} \frac{b_{\text{on}}}{T_b}$$

Shot Noises – '0' with all neighbor '1's – $B_{OFF} = 0$

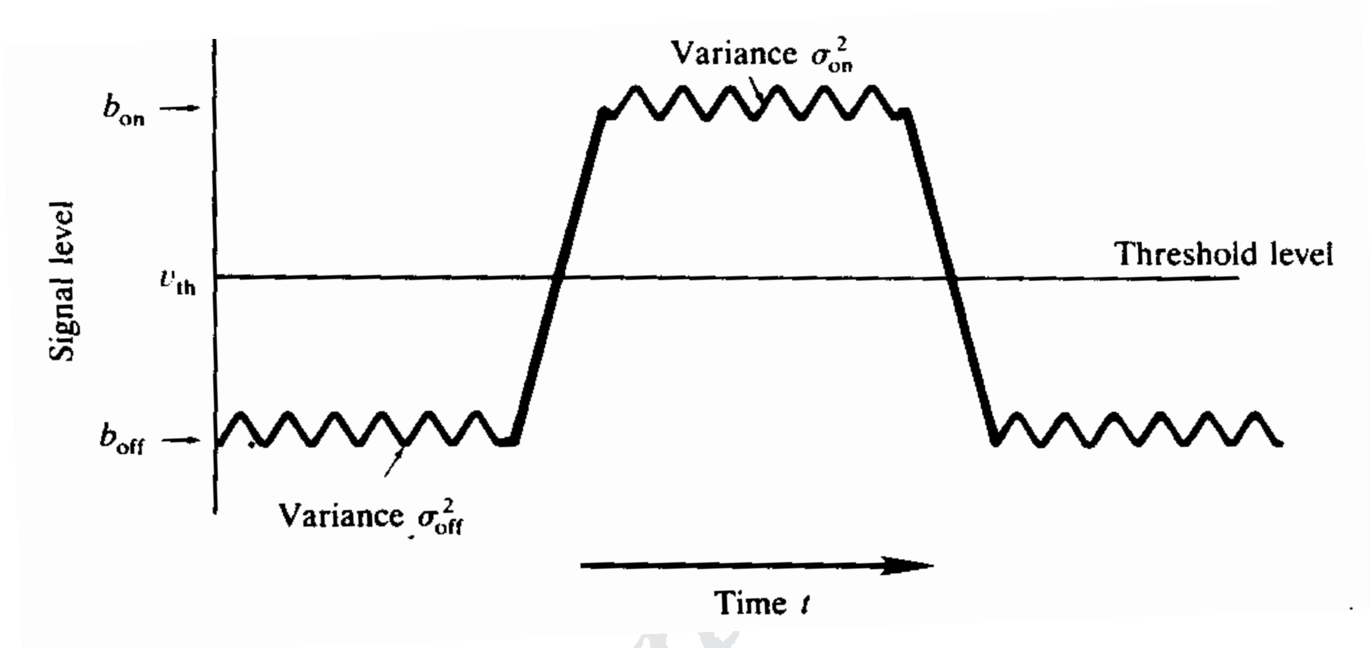
$$\langle i_0 \rangle_0 = \sum_{n \neq 0} \frac{\eta q}{h\nu} b_{on} \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} h_p(t - nT_b)\, dt$$

$$= \frac{\eta q}{h\nu} \frac{b_{on}}{T_b} \left[\sum_{n=-\infty}^{\infty} \int_{-T_b/2}^{T_b/2} h_p(t - nT_b)\, dt - \int_{-T_b/2}^{T_b/2} h_p(t)\, dt \right]$$

$$= \frac{\eta q}{h\nu} \frac{b_{on}}{T_b} (1 - \gamma) \qquad \gamma = \int_{-T_b/2}^{T_b/2} h_p(t)\, dt$$

- Substitute $<i_o>_o$ and $<i_o>_1$ in $v_s^2(t)$ to find worst case shot noise for pulse '1' and '0'.

SNR Required to achieve min BER



- Assuming output voltage is approximately Gaussian.
- Mean and variance of Gaussian output for '1' and '0' are b_{on}, σ_{on}^2 and b_{off} and σ_{off}^2.
- Decision threshold v_{th} set for equal error probability for '1' and '0'.

$$P_0(v_{th}) = P_1(v_{th}) = \frac{1}{2}P_e$$

SNR Required to achieve min BER

- Error probability for '1' and '0' are

$$P_e = \frac{1}{\sqrt{2\pi}\,\sigma_{off}} \int_{v_{th}}^{\infty} \exp\left[-\frac{(v - b_{off})^2}{2\sigma_{off}^2}\right] dv$$

$$P_e = \frac{1}{\sqrt{2\pi}\,\sigma_{on}} \int_{-\infty}^{v_{th}} \exp\left[-\frac{(-v + b_{on})^2}{2\sigma_{on}^2}\right] dv$$

- Defining Q related to SNR to achieve desired min BER-

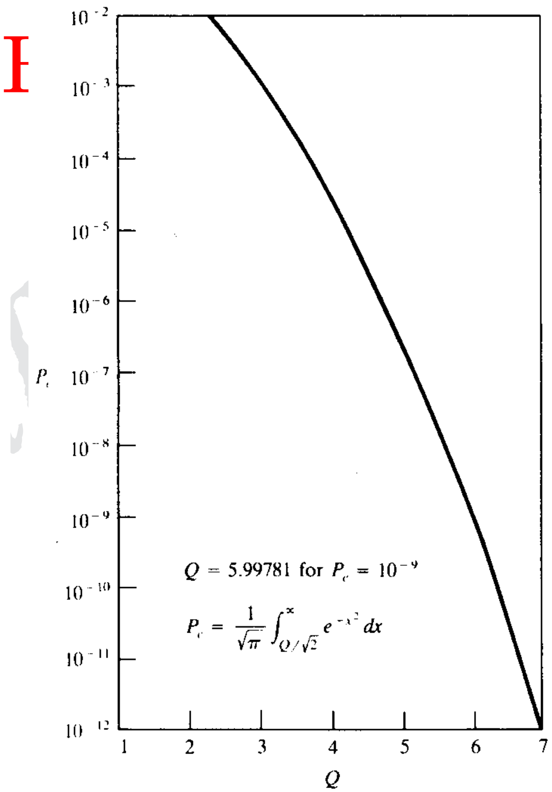
$$Q = \frac{v_{th} - b_{off}}{\sigma_{off}} = \frac{b_{on} - v_{th}}{\sigma_{on}}$$

- Putting $Q/\sqrt{2} = x$, change integral and limits.

SNR Required to achieve min I

$$P_e(Q) = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} e^{-x^2}\, dx$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{Q}{\sqrt{2}}\right)\right]$$



- Relative to noise at b_{off}, threshold v_{th} must be ATLEAST Q standard deviation above b_{off}.
- Or, v_{th} should be above b_{off} by rms value Q.
- Relative to noise at b_{on}, threshold v_{th} must not be below b_{on} by more than Q sandard deviation to have min BER

Noise Penalty in practical system- POWER Penalty

- In practical system, many factors degrade the performance.
- We assumed that –
  - Optical energy of each bit is impulse response h(t).
  - Zero energy sent during ‘0’.
  - Receiver amplifier sharply band limited.
  - No random variation in amplitude and arrival time of bit.
- In practical systems, each violation demands increase in received signal power to ensure given error probability.
- This additional excess power ΔP required in practical system is called power penalty – in dB

$$\Delta P = 10 \log \frac{b_{\text{on, nonimpulse}}}{b_{\text{on, impulse}}}$$

1. Non-Zero Extinction Ratio

- Assumed $b_{off} = 0$ during '0'.
- In actual system, light source biased slightly ON at all times to obtain shorter turn-on time in LED or keep it above threshold in LASER.
- Extinction ratio Є is optical energy emitted in the '0' pulse to that during '1' pulse.
- $Є = b_{off} / b_{on}$
- Varies between 0 and 1.
- Any dark current in photodiode appears to increase Є.
- With equally probable '0' and '1', minimum received power (sensitivity) $P_{r\ min}$ is given by average energy detected per pulse times the pulse rate $1 / T_b$.

1. Non-Zero Extinction Ratio

$$P_{r,\min} = \frac{b_{\text{on}} + b_{\text{off}}}{2T_b}$$

$$= \frac{1+\epsilon}{2T_b} b_{\text{on}}$$

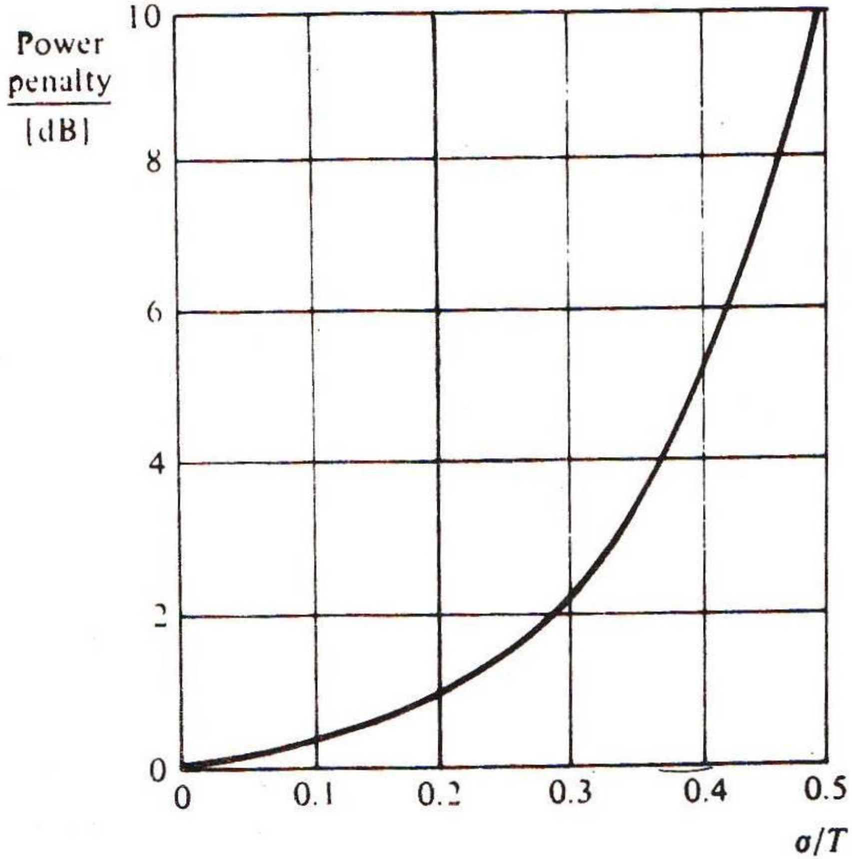
- The extinction ratio penalty i.e., the penalty in receiver sensitivity as a function of extinction ratio is-

$$y(\epsilon) = \frac{P_{r,\min}(\epsilon)}{P_{r,\min}(0)}$$

2. Finite Pulse Width and Timing Jitter

- Received optical pulse has a finite pulse width.
- Some timing jitter is present.
- Hence noise penalty is required for—
  - Non-optical filtering is needed to provide equalization against pulse distortion or to minimize ISI.
  - Some ISI remains and degrades SNR.
- To calculate magnitude of these effects, it is necessary to define
  - Shape of received pulse.
  - The distribution of the jitter
- We deal with only the former.

2. Finite Pulse Width and Timing Jitter



- Power penalty Vs ρ/T for Gaussian shaped pulse shown.
- ρ is rms width of the pulse (due to changes in pulse shape.) T is basic pulse width.

2. Finite Pulse Width and Timing Jitter

- It demonstrates possible trade-off between bit rate and signal power.
- Relates effects of fiber attenuation and fiber disperssion.
- Power penalty < 1db if ρ remains less than T/5.
- Result independent of pulse width.
- But if ρ > 1dB, PP increases sharply.
- It becomes more sensitive to pulse shape.
- In practice, system is
  - either limited by fiber dispersion (BW Limited)
  - or by fiber attenuation(Power Limited) .
  - Possible trade-off between two is quite small.

PREAMPLIFIER TYPES

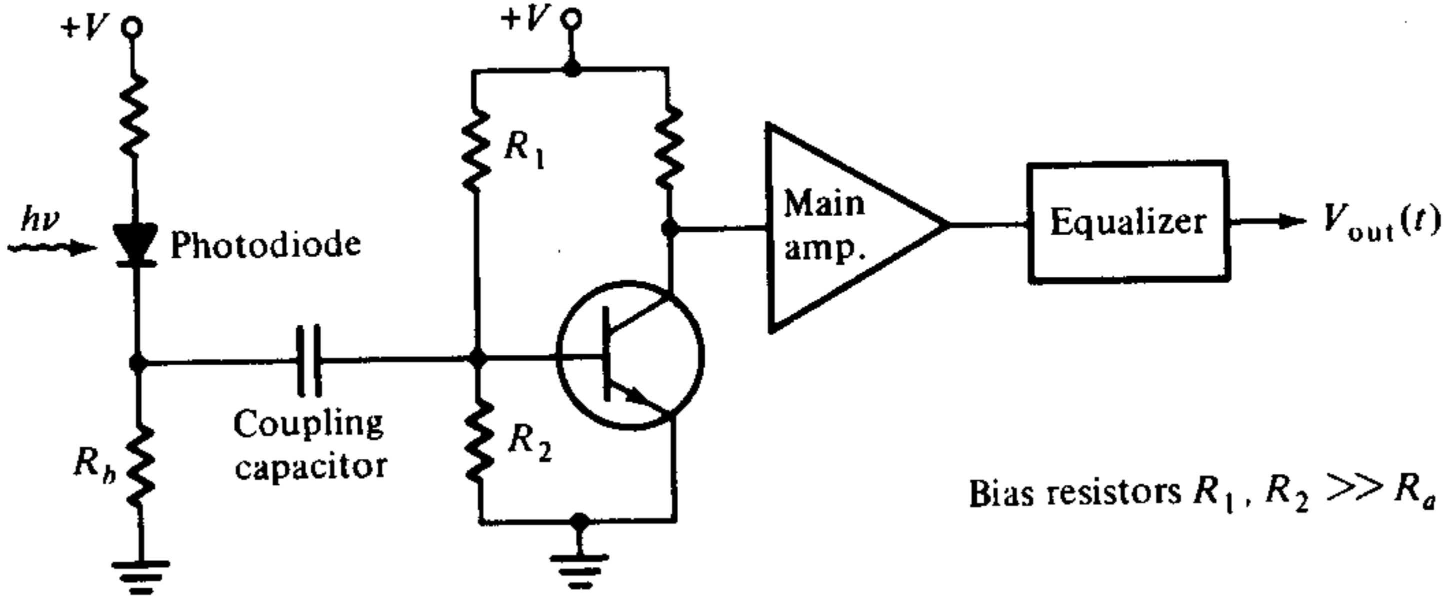
- Sensitivity and bandwidth of a receiver are effected by noise sources at the front end, i.e. at preamplifier.
- Preamplifier should give maximum receiver sensitivity with desired bandwidth.
- Three main types. But intermediate types can also be used.

PREAMPLIFIER TYPES – LOW IMPEDANCE (LZ)

- Simplest, but not optimum design.
- Photodiode operates into a low impedance amplifier (appox.50Ω)
- Bias or load resister R_b used to match amplifier impedance by suppressing standing waves and give uniform frequency response.
- R_b with amplifier capacitance gives BW equal to or greater than signal BW.
- LZ amplifier can operate over a wide BW.
- Gives low resistivity as a small voltage develops across amplifier input and R_b .
- Hence used only for short distance applications where sensitivity not of concern.

PREAMPLIFIER TYPES – HIGH IMPEDANCE (HZ)

BJT



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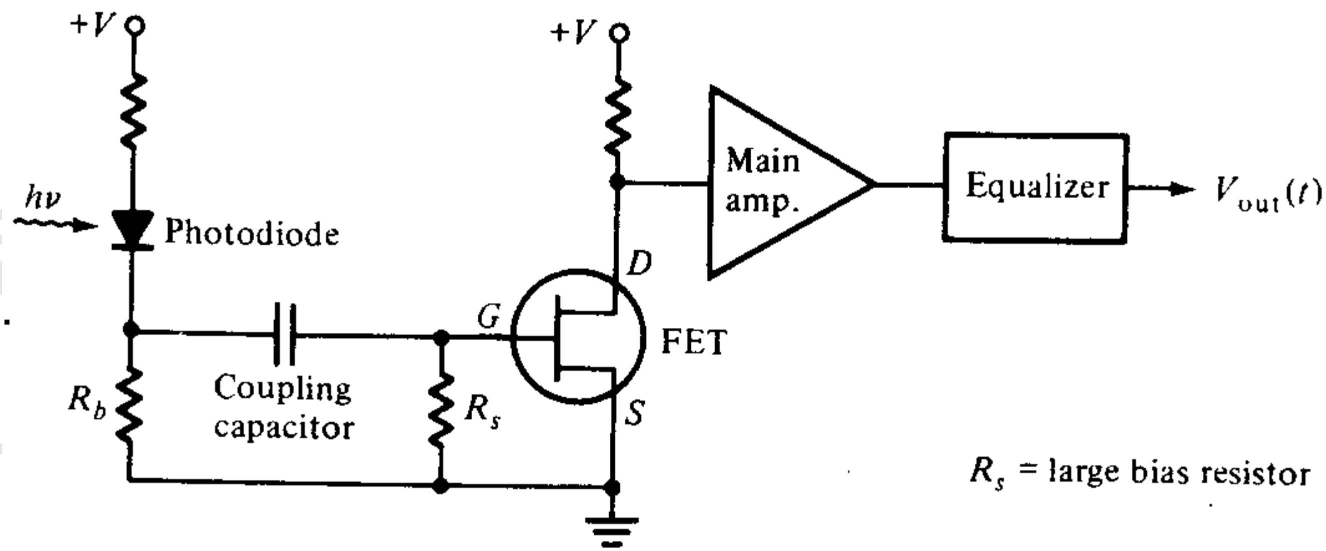


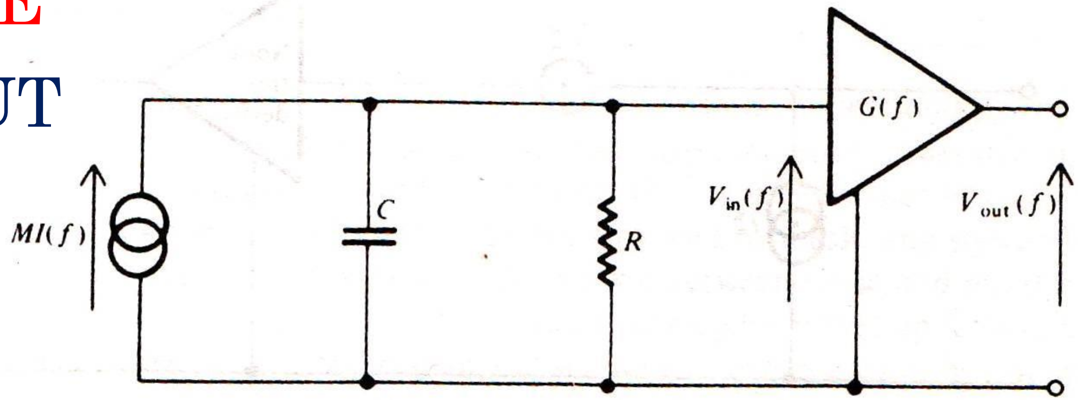
KEISER

PREAMPLIFIER TYPES – HIGH IMPEDANCE (HZ)

- Noise reduced by reducing input capacitance.
- By selecting.
  - Low capacitance, high frequency devices.
  - Detector of low dark current.
  - Bias resistor of minimum thermal noise.
- Thermal noise can be reduced by high impedance amplifier with large photo detector R_b , hence HZ amp^r.
- But causes large RC time constant – low front-end BW.
- Signal gets integrated.
- Equalization technique required. (Differentiator).
- Integrator-differentiator is core of HZ design.
- Gives low noise but low dynamic range of signal.

High impedance – Signal Output

- $R = R_a || R_b$
- $C = C_a + C_b$



$$V_{in}(f) = \frac{RMI(f)}{(1 + j2\pi fCR)}$$

$$V_{out}(f) = G(f)V_{in}(f) = G(f)\frac{RMI(f)}{(1 + j2\pi fCR)}$$

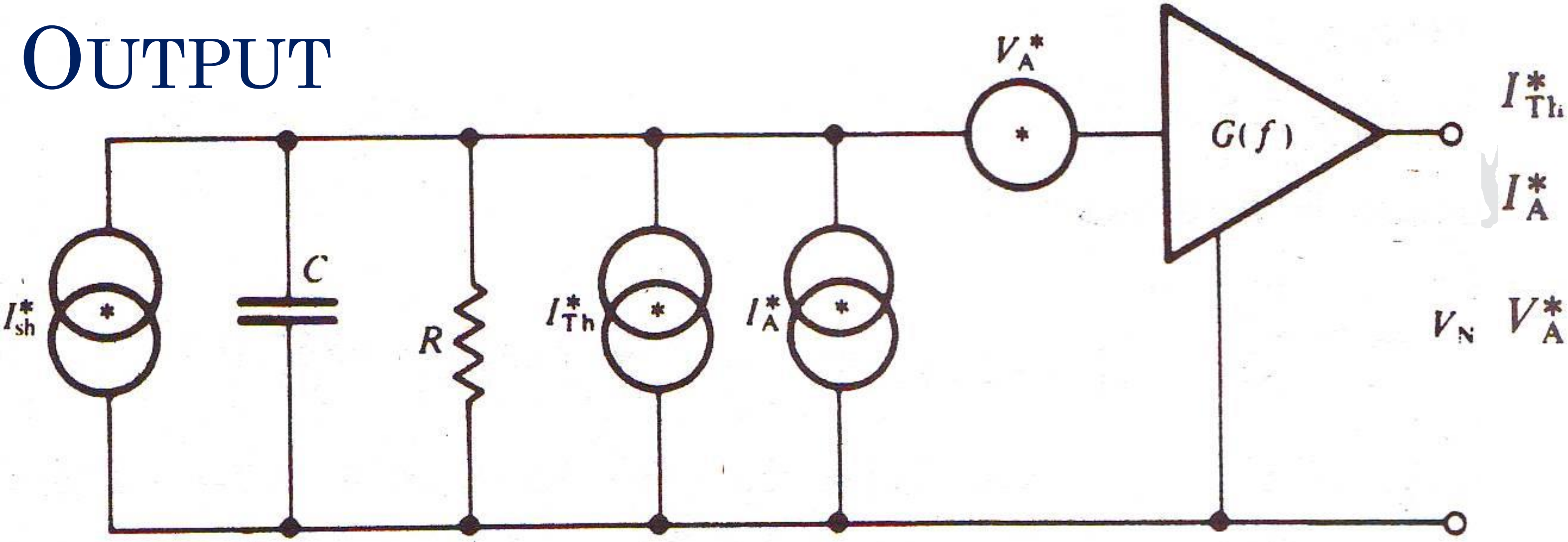
$$G(f) = G_0(1 + j2\pi fCR)$$

$$V_{out} = G_0 MRI$$

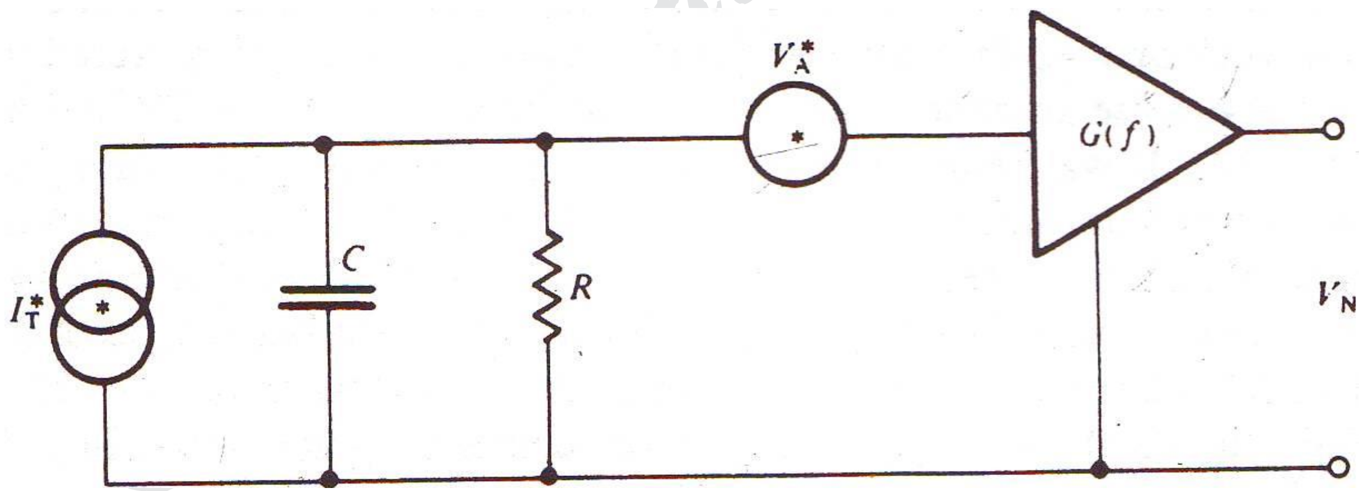
High impedance– NOISE Output

$$I^*_{sh} = (2eIM^2F)^{1/2}.$$

$$I^*_{Th} = \left(\frac{4kT}{R}\right)^{1/2}$$



$$(I^*_T)^2 = (I^*_{sh})^2 + (I^*_{Th})^2 + (I^*_A)^2 = (2eIM^2F) + (4kT/R) + (I^*_A)^2$$



HIGH IMPEDANCE– NOISE OUTPUT

$$V_N^2 = \int_{\Delta f} |G(f)|^2 (V_A^*)^2 \, df + \int_{\Delta f} \frac{|G(f)|^2 R^2 (I_T^*)^2 \, df}{|1 + j2\pi f CR|^2}$$

$$V_N^2 = G_0^2 \int_0^{\Delta f} \{(1 + 4\pi^2 f^2 C^2 R^2)(V_A^*)^2 + R^2 (I_T^*)^2\} \, df$$

$$V_N = [\{1 + (4\pi^2/3)(\Delta f)^2 C^2 R^2\}(V_A^*)^2 + R^2 (I_T^*)^2]^{1/2} (\Delta f)^{1/2}$$

$$K = \frac{V_{out}}{|V_N|} = \frac{MIR}{\{(1 + \frac{4}{3}\pi^2 (\Delta f)^2 C^2 R^2)(V_A^*)^2 + R^2 (I_T^*)^2\}^{1/2} (\Delta f)^{1/2}}$$

$$K = \frac{I}{\left\{ \frac{(V_A^*)^2}{M^2} \left(\frac{1}{R^2} + \frac{4\pi^2}{3} (\Delta f)^2 C^2 \right) + 2eIF + \frac{4kT}{M^2 R} + \frac{(I_A^*)^2}{M^2} \right\}^{1/2} (\Delta f)^{1/2}}$$

(a) (b) (c) (d) (e)

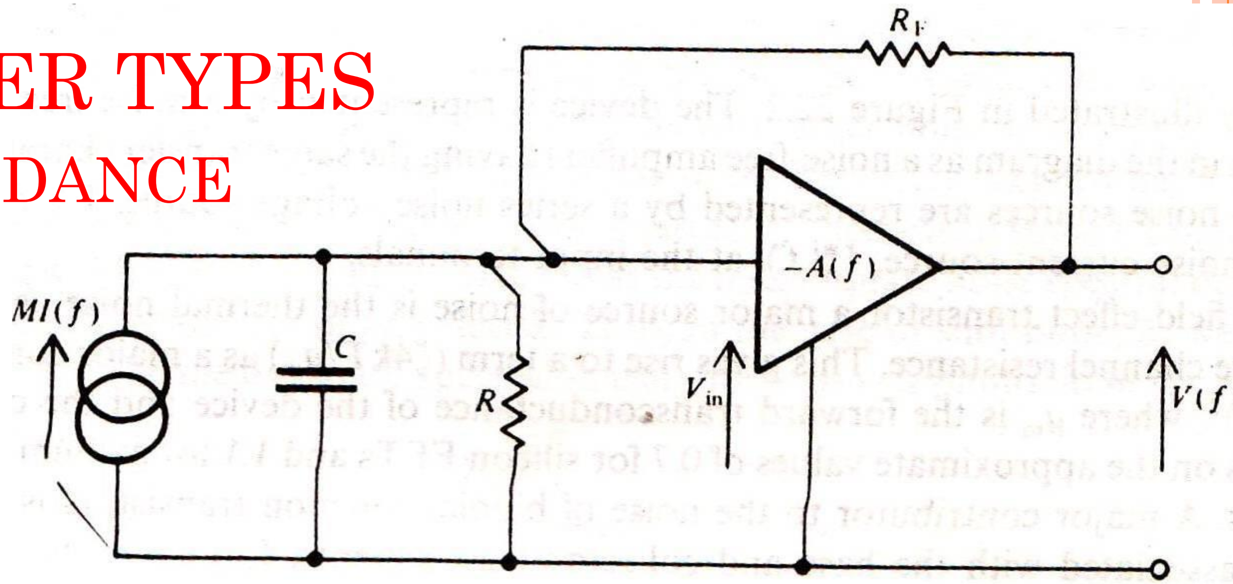
High impedance (HZ)- ANALYSIS

- SNR – K can be improved by increasing M until shot noise term (iii) increases by F(M).
- F(M) increases with M, gets comparable to other terms. Has optimum value for M.
- K improves by increasing front end R till (i) and (iv) are significant. But increases RC. Hence requires more equalization and low C.
- If equalization required, (ii) dominates at high freq. Noise increases as C^2. Hence low C required.
- Shot noise (iii) depends on input signal level.
- Assumption of all noises statistical not true in reality.
- All noises including F(M) not purely Gaussian.

HIGH IMPEDANCE (HZ)- ANALYSIS

- Hence Actual SNR may be lesser.
- Two limitations due to Integrator-differentiator
- Equalization required for broadband applications.
- limited dynamic range.

PREAMPLIFIER TYPES – TRANS IMPEDANCE



- High gain and high impedance amplifier with feedback.
- Gives low noise and large dynamic range.
- $1/R = 1/R_a + 1/R_b + 1/R_f$
- $C = C_a + C_d$

TRANS IMPEDANCE – SIGNAL POWER $V_{in} = -V/A$

$$MI + \frac{V - V_{in}}{T_F} = V_{in}\left(\frac{1}{R} + j2\pi f C\right)$$

$$MI = -V\left(\frac{1}{R_F} + \frac{1}{AR_F} + \frac{1}{AR} + \frac{j2\pi f C}{A}\right)$$

$$V = \frac{-R_F MI}{1 + \frac{1}{A} + \frac{R_F}{AR} + j\frac{2\pi f C R_F}{A}} = \frac{-R_F MI/(1 + 1/A + R_F/AR)}{[1 + j2\pi f C R_F/(1 + R_F/R + A)]}$$

$$A \gg 1 + R_F/R$$

$$V \simeq \frac{-R_F MI}{(1 + j2\pi f C R_F/A)}$$

TRANS IMPEDANCE – SIGNAL POWER

$$A \gg 2\pi C R_F \Delta f$$

$$V \simeq -R_F M I$$

TRANS IMPEDANCE – NOISE POWER

$$(V_F^*)^2 = 4kTR_F$$

$$\left(V_A^* + \frac{V_{N1}^*}{(-A)}\right)\left(\frac{1}{R} + j2\pi f C\right) = \left(V_{N1}^* - V_A^* - \frac{V_{N1}^*}{(-A)}\right)\frac{1}{R_F}$$

$$\therefore \quad V_{N1}^* = V_A^* R_F \left(\frac{1}{R_F} + \frac{1}{R} + j2\pi f C\right) \Big/ \left(1 + \frac{1}{A} + \frac{R_F}{AR} + \frac{j2\pi f C R_F}{A}\right)$$

$$\simeq V_A^* \left(1 + \frac{R_F}{R} + j2\pi f C R_F\right)$$

TRANS IMPEDANCE – NOISE POWER

For the current source

$$\left(V^*_{N2} - \frac{V^*_{N2}}{(-A)}\right)\frac{1}{R_F} + I^*_T = \frac{V^*_{N2}}{(-A)}\left(\frac{1}{R} + j2\pi fC\right)$$

$$\therefore \quad V^*_{N2}\left(\frac{1}{R_F} + \frac{1}{AR_F} + \frac{1}{AR} + \frac{j2\pi fC}{A}\right) = -I^*_T$$

$$\therefore \quad V^*_{N2} \simeq -R_F / I^*_T$$

TRANS IMPEDANCE – NOISE POWER

For the feedback source

$$V^*_{N3} = V^*_F + \frac{V^*_{N3}}{(-A)} = \frac{V^*_F}{(1 + 1/A)} \simeq V^*_F$$

$$(V^*_N)^2 = (V^*_{N1})^2 + (V^*_{N2})^2 + (V^*_{N3})^2$$

$$= (V^*_A)^2\left\{\left(1 + \frac{R_F}{R}\right)^2 + 4\pi^2 f^2 C^2 R_F^2\right\} + R_F^2(I^*_T)^2 + (V^*_F)^2$$

TRANS IMPEDANCE – NOISE POWER

$$V_N = \left[(V_A^*)^2 \left\{ \left(1 + \frac{R_F}{R}\right)^2 + \frac{4\pi^2}{3} (\Delta f)^2 C^2 R_F^2 \right\} + R_F^2 (I_T^*)^2 + 4kTR_F \right]^{1/2} (\Delta f)^{1/2}$$

$$(I_T^*)^2 = (2eIM^2F) + (4kT/R) + (I_A^*)^2$$

$K = V/V_N$

$$\frac{I}{\left[\frac{(V_A^*)^2}{M^2} \left\{ \left(\frac{1}{R} + \frac{1}{R_F}\right)^2 + \frac{4\pi^2}{3} C^2 (\Delta f)^2 \right\} + 2eIF + \frac{4kT}{M^2}\left(\frac{1}{R} + \frac{1}{R_F}\right) + \frac{(I_A^*)^2}{M^2} \right]^{1/2} (\Delta f)^{1/2}}$$

(a) (b) (c) (d) (e)

PREAMPLIFIER TYPES– TRANS IMPEDANCE - ANALYSIS

- R and R_f can be increased to reduce SNR, (i) and (iv) without equalization provided $A \gg 2\pi C R_f B$.
- It has wide dynamic range.
- Output resistance is small so that amplifier is less susceptible to pickup noise.
- Transfer characteristic is actually trans impedance feedback resister. Hence amplifier stable and easily controlled.
- Although TZ amp^r is less sensitive than HZ amp^r as $S/N_{TZ} > S/N_{HZ}$, the difference is usually only 2 to 3 dB for most practical wideband design.